The Economics of Religious Communities*

Jean-Paul Carvalho† Michael Sacks‡

Abstract

The religious club model is central to the economics of religion. To extend the scope of its application, we develop the first club model to combine (i) network externalities, (ii) discrimination against members, and (iii) competition among religious leaders. This leads to new insights into religious communities. Social integration by a religious minority depends on economic development, religious commitment, discrimination, and religious competition. These factors interact with the internal organization of the community to produce distinctive patterns of behavior. A rising share of community members with high religious commitment fractures the community, producing either assimilation by moderates or schism. Once a critical level is reached, social integration declines rapidly with religious commitment. Blanket discrimination against all community members makes the religious community stricter and more cohesive. Stigmatizing active religious participation promotes social integration on the whole, but can create an extreme isolationist sect. Religious competition reduces religious participation and boosts social integration. The results provide guidance for empirical work on religious discrimination.

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†Department of Economics and Institute for Mathematical Behavioral Sciences, University of California, Irvine, 3151 Social Science Plaza, Irvine, CA 92697, `jpcarv@uci.edu`.

‡School of Business, Clarkson University, Potsdam, New York 13699, `msacks@clarkson.edu`.

1 Introduction

The extent to which a religious minority socially integrates is of significant contemporary and historical interest. For centuries up to World War II, Jews were the largest religious minority in Europe. They faced formal and informal barriers to participation in mainstream society, which were lifted in the 19th century, a process called Jewish Emancipation. Some Jewish communities embraced the new opportunities and integrated, while others resisted (Carvalho and Koyama, 2016a; Carvalho et al., 2017). Today, Muslims are the largest religious minority in most European nations (Pew-Templeton, 2011). Their social integration has been a subject of interest for both mainstream political parties and xenophobic movements, inspiring bans on Islamic symbols such as minarets and face veils (*niqab*), influencing elections and referenda, and producing sharp political cleavages in the European Union.

To understand social integration by a religious minority, the starting point is the religious club model of Iannaccone (1992), which is central to the economics of religion (see Iyer, 2016; Carvalho, 2019, for reviews). According to this model, costly requirements in terms of dress, diet and ritual screen out free-riders and induce members to divert time and money to the group. These screening and substitution effects can raise welfare by reducing the underprovision of club goods (Aimone et al., 2013). The religious club model has been extremely fruitful, explaining education and fertility choices by ultra-orthodox Jews (Berman, 2000), the religious menus and monitoring institutions of the LDS church (McBride, 2007), and veiling practices by Muslim women (Carvalho, 2013), among many other phenomena. When it comes to social integration by a religious minority, however, the model has several limitations.

Among a set of individuals born into a religious community, we would like to analyze the tradeoff between religious participation in the community and social and economic integration. To do so requires a model with several features. First, there should be network externalities. The greater the participation by community members, the more each member benefits from participating. Second, the model must incorporate discrimination against community members in the broader society, an important factor in integration (Adida, Laitin and Valfort, 2016). Third, social integration should be compared under different competitive structures, i.e. religious monopoly versus competition among religious leaders. For example, European governments have interacted with Muslim minorities through 'community leaders' who are granted market power (Sen, 2006). We would like to know what effect such policies have on social integration. The standard religious club model has none of these features, and ours is the first to combine any of them. Building on the religious club model in this way leads to new insights into religious communities.

Social integration depends on external conditions, such as economic development and discrimination, as well as internal conditions, such as religious preferences and competition. Development (e.g. higher wages) increases integration, while high religious commitment reduces integration. The effect of discrimination against community members (e.g. social stigma or labor market discrimination) depends crucially on the nature of the discrimination. *Blanket discrimination* against all community members reduces social integration and makes the community more cohesive. In contrast, *stigma* aimed at 'actively religious' members increases social integration by community members on average, but can give rise to an extreme isolationist sect. It is such sects that are at risk of transitioning to militant activities (Berman, 2009). We also find that competition reduces religious participation, contrary to the literature on religious markets (e.g. Iannaccone, 1991). Unlike a religious monopolist who can push members to the point of indifference between joining and assimilating, competing religious leaders bid down the strictness of the club toward a welfare-maximizing level.

The effect of these factors is more complex and interesting than one might expect, because of their interaction with the internal organization of the community. For example, religious strictness is a non-monotonic function of outside economic opportunities. As outside options improve, the religious leader lowers strictness in order to retain moderate low-commitment types. At some point, however, it is not worthwhile to liberalize any further. The leader raises strictness, inducing moderates to assimilate, and caters exclusively to high-commitment types at high strictness. Similarly, a rising share of high-commitment types fragments the community, leading to either assimilation by moderates or schism. Once a critical mass of high-commitment types is reached, religious participation scales superlinearly with their share in the community. These patterns are generated by the endogenous club structure of religious communities. In Section 4, we provide guidance for empirical work on religious discrimination by showing how to account for this club structure and go beyond the simple directional predictions of existing work.

Introducing network externalities to a club model poses a number of technical challenges. First, there are multiple equilibria. To focus on interesting equilibria, we allow for coalitional deviations, which are central to the formation and fragmentation of clubs. Second, standard club models of religion assume that religious leaders always screen out low-commitment types. When there are positive network externalities, screening emerges endogenously and only under certain conditions which we characterize.[1] Third, the literature on religious competition uses Hotelling-style models in which individuals have fixed preferences over religious strictness and join the organization closest to their ideal point (Barros and Garoupa, 2002; Montgomery, 2003; McBride, 2008; Iyer et al., 2014). Positive network externalities make the problem more complex. Rather than an individual's ideal strictness being primitive, it is determined endogenously by the size of the club.

Our paper is related to the following work. Social integration of ethnic minorities has been linked to education (Constant and Zimmermann, 2008; Constant et al., 2009), labor market conditions (Bisin et al., 2011a), discrimination (Bisin et al., 2011b; Eguia, 2017), residential segregation (Bisin et al., 2016), social influence (Austen-Smith and Fryer, 2005; Carvalho and Koyama, 2016b) and community size (Lazear, 1999; Advani and Reich, 2015). When turning to religious minorities, we pay special attention to club structure, and the role of religious organizations in screening and coordinating community members. In making the distinction between stigma faced by actively religious community members and blanket discrimination against all community members, we follow Bisin et al. (2011b) who distinguish between conditional and unconditional discrimination against ethnic minorities. Finally, a companion paper, Carvalho and Sacks (2021), examines a dynamic extension of the static model presented here, with a focus on how religious leaders can radicalize a moderate community over time.

[1] Carvalho and Koyama (2016a) examine endogenous screening without network externalities or competition.

This paper is also related to the more general literature on local public and club goods with heterogeneous populations. Along with the Hotelling-style models of religious competition mentioned above, Alesina et al. (1999), Alesina and La Ferrara (2000), Haimanko et al. (2004) and Polborn (2008) present models of club and local public good provision in which preferences over location are fixed and exogenous. As noted above, our model is unique in that the ideal location of an organization from each member's perspective depends on the size and composition of its membership.

The paper is structured as follows. Section 2 models a religious community with a single religious leader. Section 3 examines the effect of free entry by competing religious leaders. Based on this model, Section 4 provides guidance for empirical work on religious discrimination. Section 5 concludes.

2 A Model of Religious Community

Consider a religious community I embedded in a large (unmodeled) society. The community has a finite set of leaders, each with their own religious organization. Individuals vary in their religious commitment. There are two types: high commitment types and low commitment types. Formally, $I = [0,1]$ is a continuum of agents with Lebesgue measure λ and Borel algebra $\mathcal{B}$ on $[0,1]$. The community is partitioned into two measurable subsets. The set of high (low) commitment types is denoted by I_H (I_L). At times, we refer to low-commitment types as 'moderates'. The share of high-commitment types in the community is $p := \lambda(I_H)$.

We begin by analyzing the monopoly case in which the community is served by a single religious organization. Religious competition is introduced in Section 3.

2.1 Religious Monopoly

There is one religious organization in the community—the club—governed by a leader. While community members cannot choose between religious organizations, they can choose how intensively to participate, if at all.

Each individual divides one unit of time between work/leisure outside the community and collective production of a religious club good in the community. We call this religious participation and define *assimilation* as zero participation. The club good can include communal prayer, leisure, festivals, welfare provision, political activism and other forms of collective action. Religious participation choices by each individual are given by the measurable mapping $i \mapsto x(i) \in [0,1]$. Participation by the community is summarized by $x = (x(i))_{i \in I}$. Total club participation is $X = \int_I x(i) d\lambda$. For expositional convenience, we write $x(i)$ as x_i when there is no confusion.

The payoff function for a club member i is

$$u_i(x) = \pi \underbrace{(1 - x_i)}_{\text{outside activity}} + \theta_i \underbrace{X^{1/2}}_{\text{club good}} . \tag{1}$$

The two terms in the payoff function are i's utility from outside activity and consumption of the club good, respectively. Each unit of time spent outside of the community yields a payoff of $\pi > 0$. The value placed by i on the club good, $\theta_i > 0$, is referred to as i's religious commitment: $\theta_i = \theta_L$ for low-commitment types and $\theta_i = \theta_H$ for high-commitment types, where $\theta_H > \theta_L > 0$.

Social and economic integration by i are both measured by $1 - x_i$, interpreted respectively as the proportion of time i spends on social interactions with outgroup members and the time i spends on work to fund private consumption. The latter makes π the real wage rate faced by club members. Assimilation is an extreme form of social integration in which an individual spends all of her time outside the community.

The club faces a severe free-rider problem in production of the club good, as agents are non-atomic. Thus rules must be set for participation. We assume the club leader imposes a minimum participation requirement s: to have access to the club good, an individual must devote at least s units of time to production of the club good. Equivalently, s is a restriction on outside activity, which can be imposed indirectly through various stigmatizing requirements in terms of dress, diet and ritual (see Iannaccone, 1992; Aimone et al., 2013) or directly by monitoring time inputs (Carvalho, 2016). We refer to s as the *strictness* of the club. If $x_i \geq s$, i is deemed a member of the club and receives the payoff described by (1). The set of club members is denoted by M. These are the members of the community who are actively religious. The community is *cohesive* if all community members are members of the club. Otherwise, it is *fragmented*.

We assume the payoff to outside activity by nonmembers is $\Delta\pi > 0$, which is also their total payoff since they are excluded from the religious club good. The relative payoff to outside activity by non-members compared to members is $\frac{\Delta\pi}{\pi} = \Delta$. This measures the value of a community member's outside option. Under the economic integration interpretation, Δ is the relative wage faced by nonmembers. This is a generalization of past work, which assumes that an individual's payoff from outside activity is independent of club membership (i.e. $\Delta = 1$). If members of the club are discriminated against, Δ is greater than one. If club members have access to a special production technology (e.g. Bernstein, 1992), Δ could be less than one.

We assume the leader's objective is to maximize total religious participation X. Barros and Garoupa (2002) assume that religious leaders maximize the welfare of their members, while McBride (2008) assumes leaders maximize membership size. Our specification combines these and other motivations. In maximizing X, religious leaders trade off membership size and participation intensity, and are pushed (endogenously) toward maximizing members' welfare under religious competition (in Section 3).

The timing of the game is as follows:

Date 0. Nature partitions the community into low and high commitment types.

Date 1. The club leader announces the minimum participation requirement s.

Date 2. Observing s, individuals simultaneously divide one unit of time between outside activity and production of a religious club good.

Date 3. The club good is produced and payoffs are received, as defined above.

Throughout, we assume that $\theta_H < 2\pi$.[2] The structure of the game is common knowledge.

[2] This allows us to ignore uninteresting boundary conditions. When $\theta_H \geq 2\pi$, the club can always demand the maximum possible time contribution ($s = 1$) by high types and get it.

2.1.1 Religious Equilibrium

Let $x(s) = (x_i(s))_{i \in I}$ and $M(s)$ describe the participation and membership strategies. Total participation is denoted by $X(s) = \int_I x(i)(s) d\lambda$. Our equilibrium concept accounts for coalitional deviations, which are central to the formation and fragmentation of clubs. We define the set of coalitions as the set of all nonempty Borel measurable subsets of I: $\mathcal{C} = \mathcal{B} \setminus \{\emptyset\}$.

Definition 1. Consider the subgame following strictness choice s. A deviation by coalition $C \in \mathcal{C}$ from $x(s)$ is an alternative profile $x'(s)$ such that $x_i(s) \neq x'_i(s)$ if and only if $i \in C$. A coalitional deviation is profitable if

$$u_i(x(s)) < u_i(x'(s)) \tag{2}$$

for all $i \in C$. The deviation is *coalitionally stable* if in addition there are no further profitable deviations from $x'(s)$ by any coalition $C' \in \mathcal{C}$.

Definition 2. The profile $x^*(s)$ implements a coalition-proof equilibrium of the subgame induced by the leader's choice of s if no coalitionally stable deviation exists.

Definition 3. A *religious equilibrium* (RE) of the game consists of a strictness choice and religious participation strategies $(s^*, x^*(s))$ in which $x^*(s)$ implements a coalition-proof equilibrium for each s and s^* maximizes the leader's payoff $X(s)$.

Coalition-proofness eliminates equilibria that arise from coordination failures, which are uninteresting in our setting. It does not, however, select a Nash equilibrium with efficient contributions to the club good (a free-rider problem), as no such Nash equilibrium exists in the absence of the minimum participation requirement s. In our setting, the notion of coalitional stability need only be applied once, unlike the recursive concept of coalition-proof Nash equilibrium introduced by Bernheim, Peleg and Whinston (1987).

When outside options are attractive, it may be impossible to avoid assimilation. Define $\underline{\Delta} \equiv 1 + \left(\frac{\theta_L}{2\pi}\right)^2$ and $\overline{\Delta} \equiv 1 + p\left(\frac{\theta_H}{2\pi}\right)^2$. Lemma 1 in the Appendix shows that when $\Delta > \underline{\Delta}$,

L types assimilate under any choice of strictness $s \in [0,1]$. When $\Delta > \max\{\underline{\Delta}, \overline{\Delta}\}$, both L and H types assimilate for all $s \in [0,1]$. Thus, assimilation is unavoidable when there is a large (common) payoff from outside activity π and/or a large relative payoff from outside activity to nonmembers Δ.

Moreover, the club leader may actually induce assimilation, when it could be otherwise avoided. The leader faces a tradeoff between forming a large club consisting of all community members at low strictness or forming a small club consisting only of high-commitment types at high strictness. We refer to the first kind of club as *inclusive* and the second as *exclusive*. When the share of high-commitment H types is sufficiently large, the leader maximizes total contributions by setting a high level of strictness, inducing assimilation by L types and extracting larger contributions from H types. L types could be induced to participate, but are *screened* out. This endogenous screening is different to standard religious club models in which screening out of low-commitment types is simply assumed.

Since agents are non-atomic, all club members devote the minimum required time s to collective production. To maximize contributions, the club leader sets strictness s as high as possible. For an inclusive club, the leader sets the maximum strictness that satisfies the low type's participation constraint IR_L up to $s = 1$:

$$\underline{s} = \begin{cases} 1 & \text{if } \Delta \leq \frac{\theta_L}{\pi} \\ \left(\frac{\theta_L}{2\pi} + \sqrt{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)} \right)^2 & \text{if } \Delta \in \left(\frac{\theta_L}{\pi}, \underline{\Delta}\right] \end{cases}. \tag{3}$$

For an exclusive club, the leader sets the maximum strictness that satisfies the high type's participation constraint IR_H up to $s = 1$:

$$\overline{s} = \begin{cases} 1 & \text{if } \Delta \leq \frac{\theta_H \sqrt{p}}{\pi} \\ \left(\sqrt{p}\frac{\theta_H}{2\pi} + \sqrt{p\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)} \right)^2 & \text{if } \Delta \in \left(\frac{\theta_H \sqrt{p}}{\pi}, \overline{\Delta}\right] \end{cases}. \tag{4}$$

If outside options are poor, in particular if $\Delta \leq (\theta_L/\pi)$, the leader can form an inclusive club at full strictness ($s = 1$). This is not possible for higher values $\Delta \in (\theta_L/\pi, \underline{\Delta}]$. In this case, the strictness of an inclusive club $\underline{s}$ is set so that IR_L binds, i.e. L types are indifferent between joining the club and assimilating. The strictness of an exclusive club is set to $\overline{s}$. An important difference is that $\overline{s}$ is increasing in the share of high-commitment types p. Given positive network externalities, the larger is p (the size of an exclusive club), the further the club leader can push members in terms of demands on their time. Note that $\overline{s} > \underline{s}$ if and only if $p > (\theta_L/\theta_H)^2$.

The types of equilibria that can arise are as follows.

Definition 4. A *cohesive equilibrium* is an RE in which $M^*(s^*) = I$, $s^* = \underline{s}$ and $x_i^* = \underline{s}$ for all $i \in I$.

Definition 5. An *exclusive equilibrium* is an RE in which $M^*(s^*) = I_H$, $s^* = \overline{s}$ and $x_i^* = \overline{s}$ for $i \in I_H$ and zero for $i \in I_L$.

We know that assimilation by all community members is unavoidable if $\Delta > \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$ (Lemma 1, Appendix). For $\Delta \leq \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$, the following proposition characterizes the set of RE.

Proposition 1. *The set of religious equilibria (RE) under monopoly is as follows.*

(i) $\Delta \leq \theta_L/\pi$*: There exists a cohesive RE with strictness* $s^* = 1$ *and contributions* $x_L^* = x_H^* = 1$.

(ii) $\Delta \in (\theta_L/\pi, \underline{\Delta}]$*: There exists a unique threshold proportion of high-commitment types* $\hat{p} \in ([\theta_L/\theta_H]^2, 1)$*, which is strictly decreasing in* Δ.

If $p \leq \hat{p}$*, there exists a cohesive RE.*

If $p \geq \hat{p}$*, there exists an exclusive RE.*

(iii) $\Delta \in \left(\underline{\Delta}, \overline{\Delta}\right]$*: There exists an exclusive RE.*

There are no other RE in these cases.

Proofs of all propositions are in the Appendix.

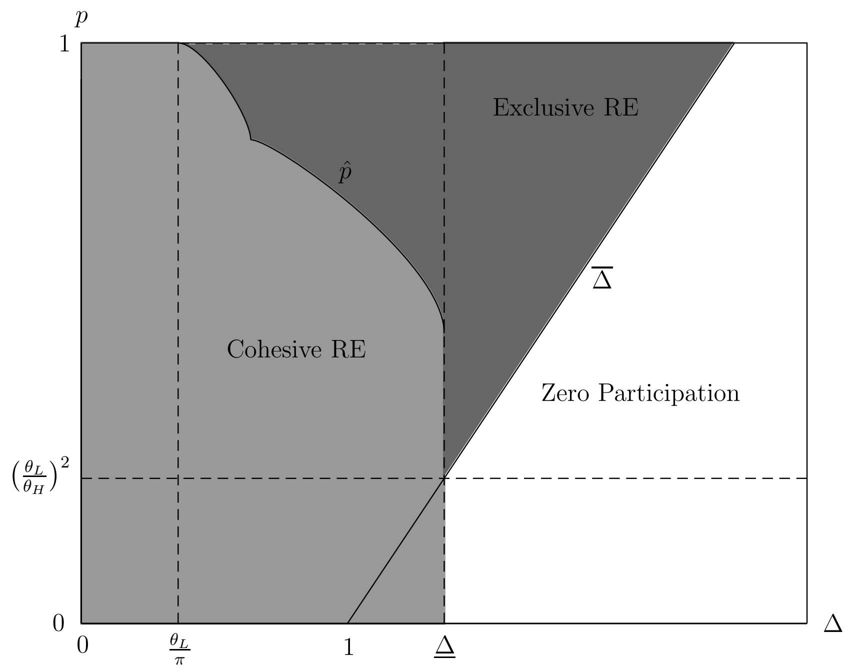


Figure 1: Equilibrium structure under monopoly.

First, note that a highly liberal club never forms: $s^* > 0$ whenever the club is nonempty, due to the need to mitigate the free-rider problem. The equilibrium structure depends on the proportion of high-commitment types p and the value of the outside option determined by π and Δ. To understand this dependence, begin by considering $\Delta \leq \underline{\Delta}$.[3] In this case, the community is cohesive if the proportion of H types is sufficiently low, $p \leq \hat{p}$. If $p > \hat{p}$, the community fragments with L types assimilating and H types forming a stricter, less integrated group. The intuition is as follows. H types value the club good more highly than L types. To induce L types to join, strictness must be set relatively low: $s \leq \underline{s}$. Alternatively, the leader could raise strictness to $\overline{s}$ and elicit larger contributions from the mass p of H types. Total participation in an inclusive club is $\underline{s}$, while for an exclusive club it is $p\overline{s}$. Hence a religious leader prefers an exclusive club when the proportion of H types is greater than the relative strictness of an inclusive club: $p > \underline{s}/\overline{s} \equiv \hat{p}$. Thus, greater religious commitment fragments a religious community.

A complete picture of the equilibrium structure is presented in Figure 1. When p is low,

[3]This condition is satisfied for a relative payoff from outside activity of $\Delta = 1$ (the standard assumption in the literature) and for some Δ greater than one.

$p \leq ([\theta_L/\theta_H])^2$, there are two possibilities. Either the community is cohesive if $\Delta \leq \underline{\Delta}$ or all community members assimilate. Otherwise, there are three possibilities. The community is cohesive if Δ is low, it fragments with the formation of an exclusive club if Δ is intermediate, and complete assimilation occurs if Δ is high ($\Delta > \overline{\Delta}$).

2.2 Comparative Statics

By Proposition 1 and equations (3)-(4), social integration is increasing in the value of outside options (π and Δ) and decreasing in religious commitment (p, θ_L, θ_H). The value of outside options in turn depends on economic development and discrimination. Taking a closer look at the effect of these factors, however, reveals a more complex and interesting picture, due to their effect on the community's internal organization. We now explore the interaction between these factors and the endogenous cohesion and fragmentation of the community.

2.2.1 Religious Commitment

Figure 2 plots total religious participation as a function of the share of extremists p in case (ii) of Proposition 1. When p is low, an inclusive equilibrium is in place. All community members choose religious participation $\underline{s}$, so total religious participation is independent of p. As the proportion of high-commitment types grows ($p \geq \hat{p}$), the community fragments and an exclusive club is formed. Both the size of this club and its participation intensity $\overline{s}(p)$ are increasing in p. Hence the religious leader benefits from greater religious commitment if and only if p is sufficiently large. Thenceforth, religious participation scales superlinearly with p. In other words, social integration begins to decline rapidly with religious commitment.

2.2.2 Economic Development & Discrimination

The religious community in our model is embedded in a larger society, characterized by a level of economic development and discrimination against members of the religious community I. By discrimination we mean any action that lowers the payoff to outside activity by members of the religious community. This includes social discrimination which negatively affects social interactions with outgroup members and labor market discrimination which lowers the expected wage faced by community members.

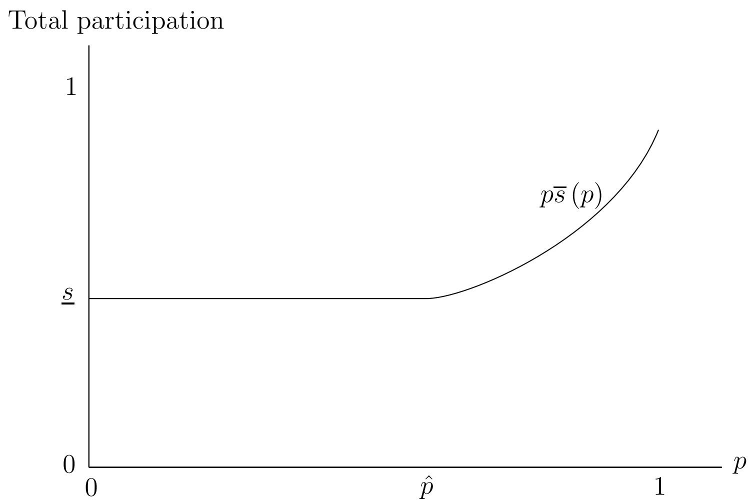


Figure 2: Total religious participation as a function of the proportion of extremists in case (ii) of Proposition 1.

Recall that π is the common component of the payoff to outside activity. Now let $\pi = w/(1+\delta)$, where w is economic development (e.g. real wage) and δ is the level of discrimination faced by all community members. We refer to δ as 'blanket discrimination' because it applies whether a community member joins the religious club or not. Following Bisin et al. (2011b), we distinguish between this and a type of conditional discrimination, which we call 'stigma'. Stigma is faced only by club members, i.e. the actively religious. Recall that the payoff to outside activity is π for club members and $\Delta\pi$ for those who assimiliate. Hence Δ is a measure of stigma.[4] It turns out that the two forms of discrimination have different effects on religious participation.

We know from Proposition 1 that total participation is $X^* = 1$ for $\Delta \leq \theta_L/\pi$ and $X^* = 0$ for $\Delta > \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$. Hence consider the intermediate range:

Proposition 2. *Suppose* $\Delta \in \left(\theta_L/\pi, \max\left\{\underline{\Delta}, \overline{\Delta}\right\}\right)$. *Total participation* X^* *is decreasing in economic development* w *and stigma* Δ, *and increasing in blanket discrimination* δ, *and strictly so whenever* $s^* < 1$.

Hence economic development and stigma reduce religious participation and increase social

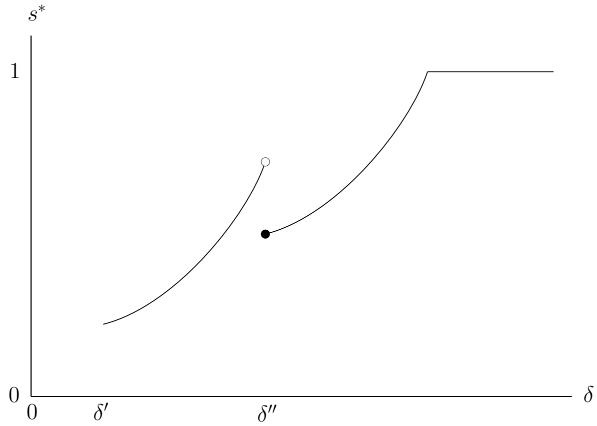
[4]For example, the real wage could be w for club members and Δw for non-members.

integration. Blanket discrimination against community members has the opposite effect. The precise relationships are again more complex than this suggests, due to their interaction with the internal organization of the community. Social integration depends on both equilibrium club membership and strictness. For p sufficiently large, there is a non-monotonic relationship between equilibrium strictness and w, δ and Δ.[5] Let us focus on the effect of discrimination. Economic development w has the same effect as stigma Δ.

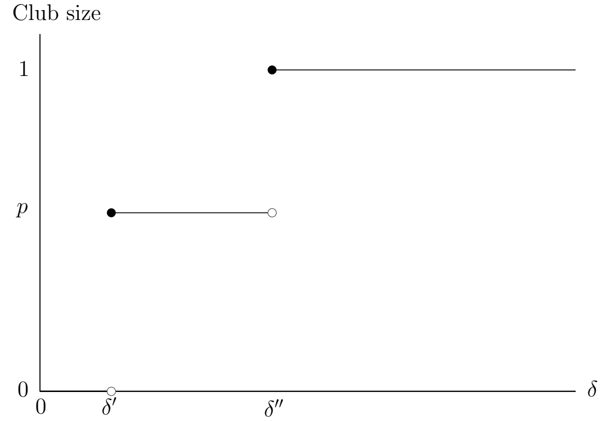
The effect of blanket discrimination on equilibrium strictness and membership, given conditional discrimination ($\Delta > 1$), is depicted by Figure 3. Recall that raising δ reduces the opportunity cost of religious participation. For δ small ($\delta < \delta'$), the opportunity cost is too high to attract moderate L types and complete assimilation is unavoidable. Instead, the religious organization forms a more extreme club composed exclusively of H types. As δ rises and the opportunity cost of religious participation falls, the organization can push H types further in terms of their participation (s^* rises). Eventually, the opportunity cost becomes low enough to make it worthwhile to attract L types and form a cohesive club. The discrete drop in strictness at δ'' to accommodate L types is shown in panel (a), while the jump in membership is shown in panel (b). Thenceforth, strictness of the club rises with blanket discrimination δ, until a cohesive community can be maintained at maximal strictness $s = 1$. In this case, we have complete social isolation of the religious community. Hence blanket discrimination inhibits social integration and produces a more cohesive and extreme religious community.

Now consider the effect of stigma Δ depicted by Figure 4. Starting from a low level of Δ, a rise in Δ makes it less attractive to join the club, prompting the club to lower strictness s^* in order to keep L types in the club. This is depicted in panel (a) by the graph up to point Δ'. Once Δ is sufficiently high, however, it is not worthwhile to liberalize any further. Instead the club benefits from raising strictness, inducing L types to exit and catering exclusively

[5] Proof available upon request. A similar pattern of non-monotonicity in δ emerges through a different mechanism in a religious club model by Carvalho and Koyama (2016a). (They only analyze δ and make the standard assumption that $\Delta = 1$.) In their model, individuals can make both time and money contributions to a club, club goods are rival, and the club does not impose a minimum contribution requirement, but rather chooses a tax on outside activity. The fact that the same result emerges under substantially different assumptions suggests there is something robust about the pattern.

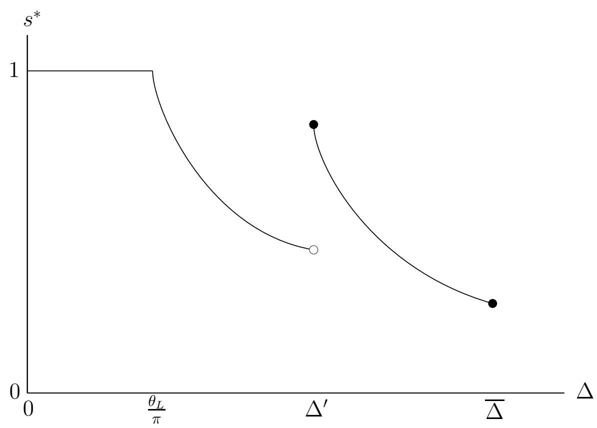


(a) Equilibrium s

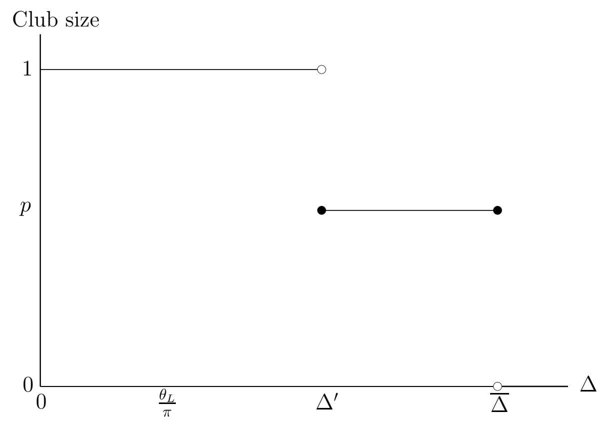


(b) Equilibrium membership

Figure 3: The effect of blanket discrimination ($\Delta > 1$)



(a) Equilibrium s



(b) Equilibrium membership

Figure 4: The effect of stigma

to H types. The discrete jump in strictness at point Δ' is shown in panel (a), while the exit of low commitment types is shown in panel (b).[6] Therefore, while stigmatizing religious participation increases social integration, it can also fragment the community, producing a small but extreme club.

As Berman (2009) shows, such clubs are at risk of being converted by militants from benign providers of club goods to violence. Due to their strictness and social isolation, radical religious clubs screen out all but the most committed types and elicit extreme contributions

[6]Zero participation is unavoidable above $\overline{\Delta}$ (Lemma 1, Appendix).

by members. This gives them an advantage in terrorism which requires costly sacrifices (e.g. suicide bombing) and safeguards against defection and infiltration (Berman and Laitin, 2008; Berman, 2009). Thus, stigmatizing religious participation as part of secularization and counter-terrorism policies could backfire.

3 Religious Competition

The religious community has so far been served by a single club. Either an individual participates in this club or is unaffiliated. Now we allow religious leaders to freely enter the market and compete for members.

There is a countable set of leaders $K = \{1, 2, \ldots\}$ indexed by k, each with her own club. We assume $|K| \geq 4$, which allows for a perfectly competitive environment. The set of 'active' clubs that attract one or more members is determined endogenously. At date 1, all leaders simultaneously announce strictness levels. The strictness profile is denoted by $\mathbf{s} = (s_1, s_2, \ldots, s_k, \ldots)$. At date 2, each agent i chooses to join one of the clubs or assimilate. If i joins club k, she also chooses her level of participation $x_{ik} \geq s_k$. The set of such individuals is denoted by M_k.

For each club k, participation is summarized by $x_k = (x_{ik})_{i \in I}$. Total participation in club k is $X_k = \int_I x_k(i) d\lambda$. The payoff function for individual $i \in M_k$ is then:

$$u_i(x_i, x_k) = \pi(1 - x_{ik}) + \theta_i X_k^{1/2}. \tag{5}$$

The payoff to assimilation is again $\Delta\pi$.

Each leader's objective is, as before, to maximize total religious participation in her club, X_k. The difference now is that the leader must compete for members. Hence the same concept of religious equilibrium in Definition 3 can be applied, with the modification that each club's strictness s_k^* maximizes X_k^* given the profile of strictness choices $\mathbf{s}^*$.

Immediately, one can point to ways in which religious competition alters the equilibrium structure. Under monopoly, the community could only fragment through assimilation by L

types. Under competition, there is a new form of fragmentation:

Definition 6. A *schismatic equilibrium* is a separating RE in which all L types join one religious club and all H types join another.

Competition also alters the strictness of religious clubs. Under religious monopoly, members were pushed to the point of indifference between joining and assimilating. Under competition, religious leaders are instead pressured to maximize members' welfare.

In a schismatic RE, each active club is homogeneous. The moderate club maximizes the welfare of its L-type members by setting strictness

$$\tilde{s}_L \equiv (1-p)\left(\frac{\theta_L}{2\pi}\right)^2. \tag{6}$$

The more extreme club maximizes the welfare of its H-type members by setting strictness

$$\tilde{s}_H \equiv p\left(\frac{\theta_H}{2\pi}\right)^2 > \tilde{s}_L. \tag{7}$$

In a cohesive RE, there is no s that maximizes the welfare of all members, since the active club's membership is heterogeneous. The club leader chooses s^* between the ideal strictness of L and H types, i.e. between

$$s_L \equiv \left(\frac{\theta_L}{2\pi}\right)^2 \text{ and } s_H \equiv \left(\frac{\theta_H}{2\pi}\right)^2. \tag{8}$$

The following proposition characterizes the set of RE for $\Delta \leq \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$. We know that complete assimilation is unavoidable for $\Delta > \max\left\{\underline{\Delta}, \overline{\Delta}\right\}$.

Proposition 3. *The set of RE under competition is as follows. In each case, $\tilde{p}$ denotes a threshold of high-commitment types which is weakly decreasing in Δ.*

Suppose $\theta_L/\theta_H \geq 1/2$.

(i) $\Delta \le \underline{\Delta}$:

If $p \le \tilde{p}$, *there exists a cohesive RE.*

If $p > \tilde{p}$, *there exists an exclusive RE.*

(ii) $\Delta \in (\underline{\Delta}, \overline{\Delta}(p)]$: *There exists an exclusive RE.*

Now suppose $\theta_L/\theta_H < 1/2$. *Define* $\tilde{\Delta} \equiv 1 + (1-p)\left(\theta_L/2\pi\right)^2 \le \underline{\Delta}$.

(iii) $\Delta \le \tilde{\Delta}$:

If $p \le \tilde{p}$, *there exists a cohesive RE.*

If $p \ge \tilde{p}$, *there exists a schismatic RE.*

(iv) $\Delta \in \left(\tilde{\Delta}, \underline{\Delta}\right]$:

If $p \le \tilde{p}$, *there exists a cohesive RE.*

If $p \ge \tilde{p}$, *there exists an exclusive RE.*

(v) $\Delta \in (\underline{\Delta}, \overline{\Delta}(p)]$: *There exists an exclusive RE.*

Strictness and participation levels are given by (6)-(8) *in the respective equilibria. There are no other types of RE in these cases.*

The equilibrium structure is more fully characterized by Figure 5.[7] When low and high commitment types are sufficiently similar ($\theta_L/\theta_H \ge 1/2$), three types of equilibria arise. Roughly speaking, when outside options are poor (Δ small), the unique RE is cohesive. When Δ is intermediate and the proportion of H types p is large, the unique RE is exclusive. L types choose to assimilate rather than form an exclusive club of their own. For large Δ, even H types assimilate.

When low and high commitment types are sufficiently distinct ($\theta_L/\theta_H < 1/2$), a fourth type of equilibrium arises. When Δ is small and the proportion of H types p is large, the unique RE is schismatic. As p is large, the H-type club sets strictness high and screens out L types.

[7] The additional details in the Figure are proved in the Appendix along with Proposition 3.

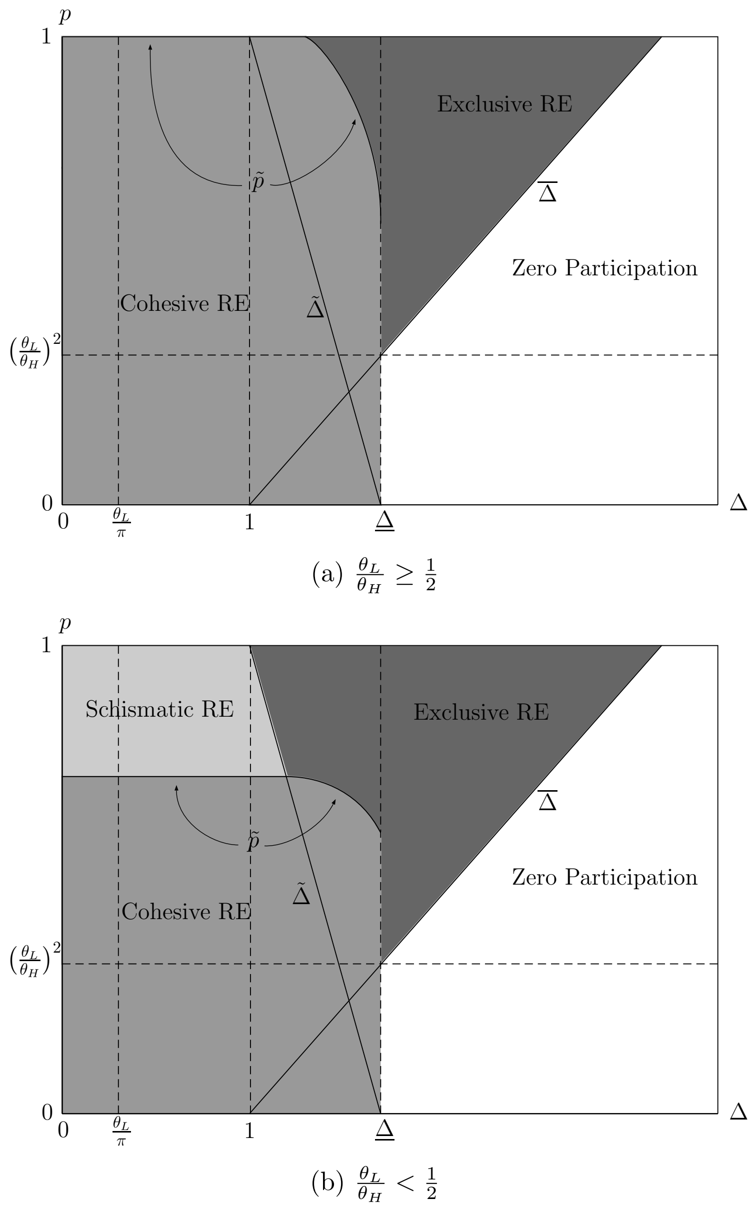


Figure 5: Equilibrium structure under competition

As outside options Δ are poor, a separate L-type club can be formed at low strictness. This schism fragments the community where it would have been cohesive under monopoly.

Despite the differences in equilibrium structure that we have described, the monopoly and competitive cases share some common features. A large share of high-commitment types p again fragments the community, either through schism or formation of a single exclusive club. Once a critical mass is reached and fragmentation occurs, religious participation scales

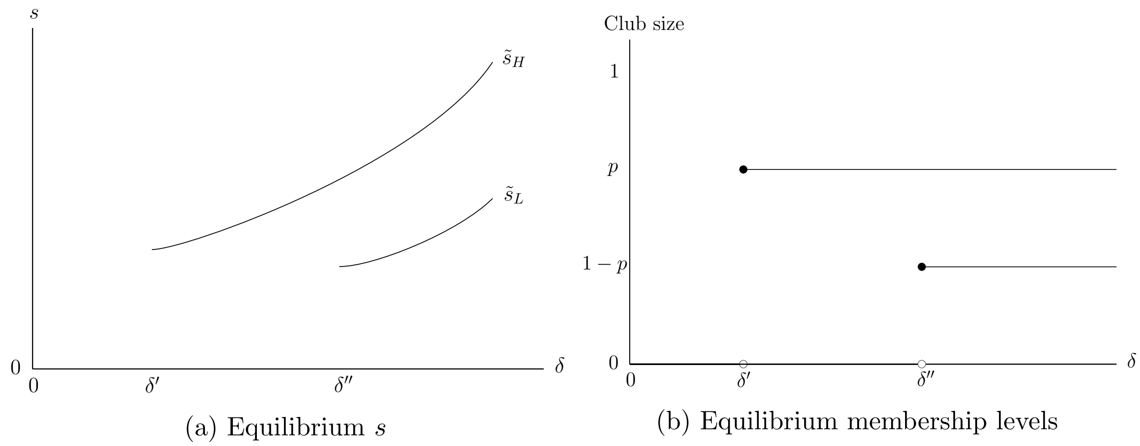


Figure 6: The effect of blanket discrimination under competition. *Case*: p large, $\frac{\theta_L}{\theta_H} < \frac{1}{2}$

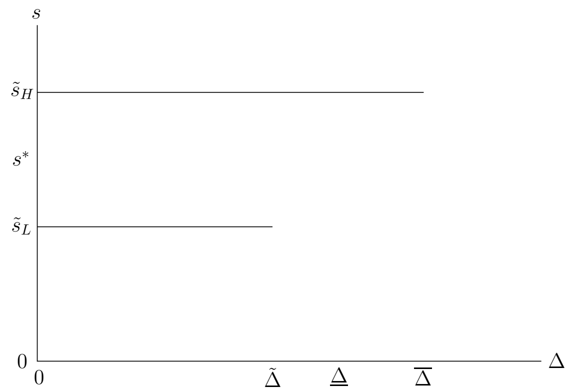
superlinearly with p. In addition, social integration is increasing in development w and stigma Δ and decreasing in blanket discrimination δ. Once again the relationship between these factors and religious strictness exhibits non-monotonicities and discontinuities, with some differences which we now describe with respect to discrimination.

3.1 The Effects of Discrimination under Competition

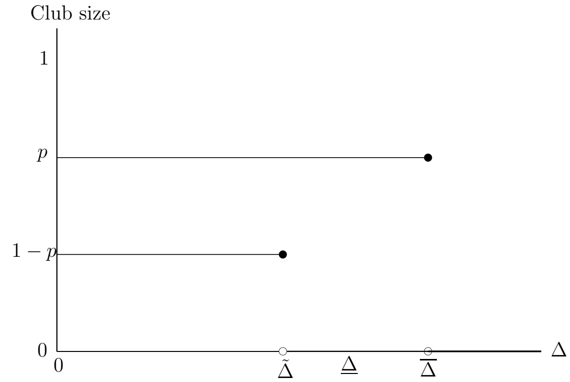
Competition alters some, but not all, of the effects of discrimination on religious participation. To see this most clearly, consider the case of p large and $\theta_L/\theta_H > 1/2$, so that a schismatic RE exists. The following relationships are derived from Proposition 3 and Figure 5.

The effect of blanket discrimination δ is depicted by Figure 6. As δ rises and the opportunity cost of participation falls, H types are *willing to* participate more intensively. That is, the welfare-maximizing strictness for H types, $\tilde{s}_H$, rises. Eventually, the opportunity cost becomes low enough to make it worthwhile for L types to form their own exclusive club at lower strictness $\tilde{s}_L$. The formation of an exclusive L type club at δ'' appears in panel (b). Thenceforth, the strictness levels of both exclusive clubs rise with δ. As under monopoly, blanket discrimination raises participation.

The effect of stigma Δ is depicted by Figure 7. Consider the extensive margin. For Δ small, a schismatic equilibrium arises. At $\tilde{\Delta}$, assimilation becomes sufficiently attractive



(a) Equilibrium s



(b) Equilibrium membership levels

Figure 7: The effect of conditional discrimination under competition. *Case*: p large, $\frac{\theta_L}{\theta_H} < \frac{1}{2}$

for L types and the exclusive L type club dissolves. Note that $\tilde{\Delta} < \underline{\Delta}$, so this requires a lower level of stigma than under monopoly. At some $\Delta \in (\tilde{\Delta}, \overline{\Delta})$, even H types choose not to participate. Again this requires, a lower level of Δ than under monopoly. Hence stigma promotes social integration at the extensive margin even more effectively than under monopoly. The same is not true of the intensive margin. Under monopoly, identity-based organizations pushed members to the point of indifference between joining and not joining. This point was determined by Δ. In contrast, competition aligns strictness choices with members' preferences, which take into account the cost of participation, so the participation constraint almost nowhere binds. Hence stigma has no effect on participation at the intensive margin under competition.

3.2 Comparing Religious Monopoly and Competition

Beginning with Adam Smith's theory of religious organization in the Wealth of Nations (Smith, 2003), the religious markets literature predicts that religious competition promotes religious affiliation and participation (see Iannaccone, 1991). It does so by increasing the quality of religious goods provided, the efficiency of production, and the fit with heterogeneous religious preferences through product differentiation. Competition also limits political distortion of religious doctrine and institutions. These are important points, supported by evidence of a positive association between religious competition and religiosity across nations

(Iannaccone, 1991; Stark and Finke, 2000). For example, it is consistent with high levels of religiosity in the United States and the decline of state religions in the United Kingdom and Sweden.

However, some of the most religious nations, such as Ireland and Poland, have very little religious competition. The religious markets hypothesis leaves out a crucial factor which our model captures. To compare levels of religious participation and social integration under religious monopoly and competition, recall that X^* is total religious participation in equilibrium. Under competition, we need to sum over all clubs, so $X^* = \sum_{k \in K} X_k^*$. The degree of social integration is simply the share of time spent outside the community, $1 - X^*$. We find the following:

Proposition 4. *Religious participation X^* is lower and social integration higher under religious competition than monopoly, and strictly so for an open set of parameters including $(\underline{\Delta}, \overline{\Delta}]$.*

This contrasts with the main prediction of the religious markets literature. The missing factor that drives our result, to which we have already alluded, is as follows. In our model, religious organizations maximize religious participation, not members' welfare. That is, they do not internalize the cost of participation by members and hence set strictness higher than the welfare-maximizing level from the perspective of members. Competitive pressures force religious organizations to lower strictness toward the welfare-maximizing level. The proof is, however, more involved than this simple intuition suggests, again because of the effect on the community's internal organization. Consider the possibility of schism. An H-type club is stricter under monopoly than under competition. However, when including the schismatic L-type club, it could be that overall religious participation is higher under competition. We find otherwise.

4 Lessons for Empirical Studies of Religious Discrimination

Existing studies of ethnic and religious discrimination find mixed evidence about the relationship between discrimination and social integration. Consider the rise in discrimination against US Muslims following 9/11. Kaushal et al. (2007) find that weekly earnings of Muslim and Arab men fell by between nine and eleven percent, despite no change in their employment rates and hours worked. Gould and Klor (2016) show that 9/11 reduced social integration by US Muslims. In states that experienced the largest rise in hate crimes, US Muslims exhibited higher rates of homogamy and fertility, and lower English language proficiency and female labor force participation. Other episodes of discrimination have produced the opposite effect, boosting social integration. For example, Fouka (2019) find that where anti-German sentiment was greatest during World War II, Germans in the US were more likely to Americanize their names and those of their children, and filed more petitions for naturalization.

These contrasting results can be reconciled by the distinction between conditional and unconditional discrimination made by Bisin et al. (2011b). In our model, blanket religious discrimination acts as a tax on outside activity and reduces social integration, whereas religious stigma acts as a tax on inside activity and raises social integration. This distinction also provides an economic explanation for puzzling patterns of behavior such as the supposed increase in veiling among Muslim women following 9/11 (Haddad, 2007), despite the greater stigma associated with it. The cost of veiling depends on the difference between the stigma faced by a veiled Muslim woman and the (blanket) discrimination faced by an unveiled Muslim woman. Thus, veiling could increase after 9/11 if stigma was swamped by a rise in blanket discrimination. More generally, the response to discrimination in terms of social integration tells us something about the mix of conditional and unconditional discrimination taking place.

By accounting for the internal organization of religious communities, our model goes beyond these simple directional predictions and suggests ways in which to advance empirical research on religious discrimination.

1. *Distributional Properties.* The empirical literature focuses on the relationship between discrimination and the *mean* degree of social integration by members of a minority. According to our approach, this discards useful information. Blanket discrimination should not only decrease social integration and increase religious participation, but should also produce convergence across community members. In particular, the least religious individuals should respond to blanket discrimination by increasing religious participation and integrating less. The opposite occurs with stigma, which we expect to polarize the community. While overall religious participation falls with stigma, this comes mainly from the left tail of the distribution, as types with low religious commitment begin to assimilate and become altogether non-religious. In contrast, the types with high religious commitment may actually increase religious participation. These distributional changes create an additional signature which distinguishes blanket discrimination from stigma. The response to discrimination by low-commitment types is key.

2. *Interaction Effects.* Similarly, any individual- or group-level characteristic that affects returns to social integration should be interacted with the response to discrimination. A larger response is expected for individuals/groups with higher returns to integration (e.g. high wages). Elsayed and De Grip (2018) examine the effect of discrimination against Muslim immigrants to the Netherlands following terrorist attacks in Europe. The find a negative impact on social integration which is most pronounced for the highly educated, employed, and less religious. Our model would identify this as evidence of blanket discrimination against Muslims, rather than stigma directed toward actively religious Muslims. This interaction effect also suggests that religious clubs play an important role in Muslim communities, reshaping their internal organization in response to discrimination.

3. *Competitive Environment.* Blanket discrimination produces different patterns of religious participation depending on the competitive structure. A religious leader may have a virtual monopoly in the community when entry costs for competing leaders are high or the government grants the leader special status. Under religious monopoly, a

rise in blanket discrimination will induce individuals with previously low levels of religious participation to join strict groups. Thus the community becomes more cohesive. Under religious competition, there is less convergence in religious participation, as previously unaffiliated individuals can form new, less strict groups. Thus, the community might remain fragmented, while being more religious.

5 Conclusion

This paper examines the economics of religious communities, showing how to model a religious community with (1) network externalities, (2) discrimination, and (3) religious competition. It makes a number of technical contributions to the religious clubs literature and produces new insights into the social integration and internal cohesion of religious communities. Social integration is increasing in economic development and stigma, and decreasing in blanket discrimination and religious commitment. These factors interact with the organization of the community to produce distinctive patterns. Once a critical mass of high-commitment types is reached, the community fragments and religious participation scales superlinearly with their share in the community. Blanket discrimination and stigma have different effects on community structure, but both produce some form of religious extremism. In contrast, religious competition moderates religious participation. We encourage empirical work on religious discrimination that takes seriously the organizational structure of religious communities. Our results provide some guidance for such work.

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Appendix

Lemma 1. *Club membership is determined by the value of the outside option as follows:*

(i) *There exists an $s \in [0,1]$ such that $M^*(s) = I$ only if*

$$\Delta \leq \underline{\Delta} \equiv 1 + \left(\frac{\theta_L}{2\pi}\right)^2. \tag{IR_L}$$

Otherwise, L types assimilate: $M^*(s) \cap I_L = \emptyset$ for all $s \in [0,1]$ in every religious equilibrium (RE).

(ii) *There exists an $s \in [0,1]$ such that $I_H \subseteq M^*(s)$ only if*

$$\Delta \leq \max\{\underline{\Delta}, \overline{\Delta}\}, \text{ where } \overline{\Delta} \equiv 1 + p\left(\frac{\theta_H}{2\pi}\right)^2. \tag{IR_H}$$

Otherwise, all types assimilate: $M^*(s) = \emptyset$ for all $s \in [0,1]$ in every RE.

Proof. (i) Consider a cohesive equilibrium. Because agents are nonatomic, $x_i(s) = s$ for all i.

The payoff to i from joining this inclusive club is

$$\pi(1-s) + \theta_i (s)^{1/2}, \tag{9}$$

which is maximized at

$$s = \left(\frac{\theta_i}{2\pi}\right)^2, \tag{10}$$

yielding a maximum of

$$\pi + \frac{1}{\pi}\left(\frac{\theta_i}{2}\right)^2. \tag{11}$$

Hence there exists an s such that $M^*(s) = I$ only if the non-participation payoff $\Delta\pi$ exceeds (11) for L types, or

$$\Delta \leq \underline{\Delta} \equiv 1 + \left(\frac{\theta_L}{2\pi}\right)^2. \tag{12}$$

(ii) Consider an exclusive equilibrium. Because agents are nonatomic, $x_i(s) = s$ for all $i \in I_H$ and $x_i(s) = 0$ for all $i \in I_L$.

The payoff to i from joining this exclusive club is

$$\pi(1-s) + \theta_H (ps)^{1/2}, \tag{13}$$

which is maximized at

$$s = p\left(\frac{\theta_H}{2\pi}\right)^2, \tag{14}$$

yielding a maximum of

$$\pi + \frac{p}{\pi}\left(\frac{\theta_H}{2}\right)^2. \tag{15}$$

Hence there exists an s such that $M^*(s) = I_H$ only if the non-participation payoff $\Delta\pi$ exceeds (15), or

$$\Delta \geq \overline{\Delta} \equiv 1 + p\left(\frac{\theta_H}{2\pi}\right)^2. \tag{16}$$

□

Proof of Proposition 1

Proof. The religious leader maximizes total participation X, so $s \in \{\underline{s}, \overline{s}(p)\}$ in equilibrium. Otherwise, the club could increase strictness (and participation) without a decline in membership.

By the same argument in Lemma 1, $x_i = s^* \in \{\underline{s}, \overline{s}\}$ for all $i \in M^*(s^*)$.

Now consider membership choices. Due to increasing returns, either all L types join or none do. Likewise for H types. As $\theta_H > \theta_L$, if all L types join, so do all H types. Hence $M^*(s) \in \{\emptyset, I_H, I\}$ and we need only consider coalitional deviations by coalitions $\mathcal{C} \in \{I_L, I_H, I\}$, as a profitable coalitional deviation by some $C \subset I_L$ implies a profitable coalitional deviation for all $i \in I_L$ (similarly for I_H and I).

Suppose there exists an s' such that $M^*(s') \neq \emptyset$. The club will never set s such that $M^*(s) = \emptyset$, as $G(s) = 0$ in this case, a minimum of its objective function. Combining this fact with Lemma 1, $M^*(s^*) = \emptyset$ if and only if $\Delta > \max\{\underline{\Delta}, \overline{\Delta}\}$.

Now suppose that $\Delta \leq \max\{\underline{\Delta}, \overline{\Delta}\}$, so that $M^*(s^*)$ equals I_H or I. We have established that $s^* = \overline{s}$ in the first case and $s^* = \underline{s}$ in the second case.

Given $x_i = s$ for all $i \in M(s)$, the club prefers to be inclusive if and only if

$$\underline{s} \geq p\overline{s}. \tag{17}$$

<u>Case 1:</u> $\Delta \leq \theta_L/\pi$ or $p \leq (\theta_L/\theta_H)^2$. By (3) and (4), $\underline{s} \geq \overline{s}$. Hence all types are willing to join the club at $s = \underline{s}$ and (17) is satisfied, so the club prefers to be inclusive.

<u>Case 2:</u> $\Delta > \theta_L/\pi$ and $p > (\theta_L/\theta_H)^2$. First, $\underline{s} = \overline{s}$ for $p = (\theta_L/\theta_H)^2$ by (3) and (4). Hence (17) holds. Second, $\underline{s} < \overline{s} = p\overline{s}$ for $p = 1$. Third, $\overline{s}$ is increasing in p and $\underline{s}$ is independent of p. Therefore, there exists a unique $\hat{p} \in \left([\theta_L/\theta_H]^2, 1\right)$, at which (17) binds. The club prefers to be inclusive if and only if $p \leq \hat{p}$. By Lemma 2 below, $\hat{p}$ is strictly decreasing in Δ.

Now we check incentive compatibility. By the construction of (3), an inclusive club can be implemented at $\underline{s}$ if and only if $\Delta \leq \underline{\Delta}$ (IR_L).

For an exclusive club to be incentive compatible at $\overline{s}$, $\Delta \leq \overline{\Delta}$ (IR_H). In addition, there must be no profitable coalitional deviation by low types. The most profitable involves I_L joining to form an inclusive club, and contributing $x_i = \overline{s}$. But $\underline{s}$, the maximum strictness L types would tolerate in an inclusive club, is less than the club's strictness $\overline{s}$ for $p > (\theta_L/\theta_H)^2$. Therefore, an exclusive club is incentive compatible when $\Delta \leq \overline{\Delta}$ and $p > (\theta_L/\theta_H)^2$.

We know the club prefers being exclusive to zero participation. Therefore an exclusive club forms if $\Delta \in \left(\underline{\Delta}, \overline{\Delta}\right]$. This establishes the proposition. $\square$

Lemma 2. *$\hat{p}$ is strictly decreasing in Δ on the domain $\left(\frac{\theta_L}{\pi}, \underline{\Delta}\right]$.*

Proof. By definition, $\hat{p}\overline{s}(\hat{p}) = \underline{s}$.

Suppose $\overline{s}(\hat{p}) = 1$. Then

$$\frac{d\hat{p}}{d\Delta} = \frac{d\underline{s}}{d\Delta},$$

which is negative for $\Delta > \theta_L/\pi$ by inspection of (3).

Now suppose $\overline{s}(\hat{p}) < 1$. Then

$$\begin{aligned}
\frac{d\hat{p}}{d\Delta}\overline{s} + \hat{p}\left[\frac{\partial \overline{s}}{\partial \Delta} + \frac{\partial \overline{s}}{\partial \hat{p}}\frac{d\hat{p}}{d\Delta}\right] &= \frac{d\underline{s}}{d\Delta} \\
\frac{d\hat{p}}{d\Delta}\left[\overline{s} + \hat{p}\frac{\partial \overline{s}}{\partial \hat{p}}\right] &= \frac{d\underline{s}}{d\Delta} - \hat{p}\frac{\partial \overline{s}}{\partial \Delta} \\
&= \frac{\hat{p}\sqrt{\overline{s}}}{\sqrt{\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)}} - \frac{\sqrt{\underline{s}}}{\sqrt{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)}},
\end{aligned} \tag{18}$$

so that $d\hat{p}/d\Delta$ is negative if and only if the RHS of (18) is negative. As $\hat{p} = \underline{s}/\overline{s}$, (18) is equivalent to

$$\begin{aligned}
\hat{p} &< \sqrt{\hat{p}}\sqrt{\frac{\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)}{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)}} \\
\hat{p} &< \frac{\hat{p}\left(\frac{\theta_H}{2\pi}\right)^2 - (\Delta - 1)}{\left(\frac{\theta_L}{2\pi}\right)^2 - (\Delta - 1)}.
\end{aligned} \tag{19}$$

Since $\hat{p} \in \left([\theta_L/\theta_H]^2, 1\right)$, (19) is satisfied. $\square$

Proof of Proposition 2

Proof. For $\Delta \in (\theta_L/\pi, \underline{\Delta})$, $X^* = \max\{\underline{s}, p\overline{s}\}$ by Proposition 1(ii). For $\Delta \in \left[\underline{\Delta}, \overline{\Delta}\right)$, $X^* = p\overline{s}$ by Proposition 1(iii).

By (3) and (4), whenever less than one, $\underline{s}$ is a continuous and strictly increasing function of δ and a continuous and strictly decreasing function of both w and Δ. The same applies to $\overline{s}$. The result follows. $\square$

Proof of Proposition 3

Proof. We shall establish the proposition by identifying the conditions under which each class of RE exists for a given p and π. We make use of the fact that x^*_{ik} equals s_k or zero for each individual $i \in I$ and club $k \in K$.

Define $\tilde{\Delta} \equiv 1 + (1-p)\left(\theta_L/2\pi\right)^2 \leq \underline{\Delta}$. It is straightforward to show that if $\Delta < \tilde{\Delta}$, L types prefer schism to zero participation. Therefore, the RE is either cohesive or schismatic. If $\Delta > \tilde{\Delta}$, L types prefer zero participation to schism. Therefore, the RE is either cohesive or exclusive.

Cohesive RE. Let $s^* \in [s_L, s_H]$ be the strictness of the unique active group. The equilibrium payoff to i is:

$$\pi\big(1-s^*\big) + \theta_i\big(s^*\big)^{1/2}. \tag{20}$$

Due to increasing returns, if there is a profitable deviation by a subset of type ω agents, then there is an even more profitable deviation by the full set of type ω agents I_ω, $\omega = L, H$. Thus only three types of deviations need to be ruled out: (I) another club attracts all agents to form a new inclusive club, (II) at least one other club forms an exclusive club by attracting all individuals of type ω only, and (III) at least one type ω chooses zero participation.

To be profitable, a type-I deviation requires there be an $s \in [0,1]$ such that:

$$\pi\big(1-s^*\big) + \theta_L\big(s^*\big)^{1/2} \; < \; \pi\big(1-s\big) + \theta_L s^{1/2}, \tag{21}$$

and

$$\pi\big(1-s^*\big) + \theta_H\big(s^*\big)^{1/2} \; < \; \pi\big(1-s\big) + \theta_H s^{1/2}. \tag{22}$$

Let $s^* \in [s_L, s_H]$ (the interval is defined by (8)). Because the RHS of (21) is strictly concave and maximized at s_L, (21) is violated for $s \geq s^*$. Because the RHS of (22) is strictly concave and maximized at s_H, (22) is violated for $s \leq s^*$. If $s^* \notin [s_L, s_H]$, both (21) and (22) hold. Hence no such deviation is profitable if and only if $s^* \in [s_L, s_H]$.

Now consider a type-II deviation to an exclusive group. The most a competing entrepreneur can do to attract L types is to set $s = \tilde{s}_L$ (defined by (6)), which yields

$$\max_{s\in[0,1]} \; \pi\big(1-s\big) + \theta_L\big((1-p)s\big)^{1/2} \; = \pi\left[1 + (1-p)\left(\frac{\theta_L}{2\pi}\right)^2\right].$$

The most a competing entrepreneur can do to attract H types is to set $s = \tilde{s}_H$ (defined by (7)), which yields

$$\max_{s\in[0,1]} \; \pi\big(1-s\big) + \theta_H\big(ps\big)^{1/2} \; = \pi\left[1 + p\left(\frac{\theta_H}{2\pi}\right)^2\right].$$

Hence the following conditions rule out a profitable type-II deviation:

$$\pi\big(1-s^*\big) + \theta_L\big(s^*\big)^{1/2} \geq \pi\left[1 + (1-p)\left(\frac{\theta_L}{2\pi}\right)^2\right], \tag{23}$$

$$\pi\big(1-s^*\big) + \theta_H\big(s^*\big)^{1/2} \geq \pi\left[1 + p\left(\frac{\theta_H}{2\pi}\right)^2\right]. \tag{24}$$

The following two participation constraints rule out a profitable type-III deviation:

$$\pi\big(1-s^*\big)+\theta_L\big(s^*\big)^{1/2}\geq \Delta\pi \tag{25}$$

$$\pi\big(1-s^*\big)+\theta_H\big(s^*\big)^{1/2}\geq \Delta\pi. \tag{26}$$

Case 1: $\Delta \leq \underline{\Delta}$ and $p \leq (\theta_L/\theta_H)^2$. $\underline{\Delta}\pi$ is an upper bound on the RHS of conditions (23)-(26). The LHS of (23) is a lower bound on the LHS of conditions (23)-(26). Therefore, it is sufficient to show there exists $s^* \in [s_L, s_H]$ such that

$$\begin{aligned}\pi\big(1-s^*\big)+\theta_L\big(s^*\big)^{1/2} &\geq \underline{\Delta}\pi \\ &= \max_{s\in[0,1]}\ \pi\big(1-s\big)+\theta_L s^{1/2}.\end{aligned} \tag{27}$$

Hence (27) is satisfied for $s^* = s_L$ and there exists a cohesive RE.

Case 2: $\Delta \leq \underline{\Delta}$ and $p > (\theta_L/\theta_H)^2$. First, note that (26) is satisfied strictly whenever (25) is satisfied. Hence it is sufficient to show there exists $s^* \in [s_L, s_H]$ that satisfies (23)-(25).

Case 2a: $\Delta \leq 1 + (1-p)\,[\theta_L/(2\pi)]^2$. In this case, the RHS of (23) is no less than the RHS of (25), so the relevant constraints are (23) and (24). Denote the smallest s^* that satisfies (24) by z_H. We have:

$$z_H = \left(\frac{\theta_H}{2\pi}(1-\sqrt{1-p})\right)^2.$$

Denote the largest s^* that satisfies (23) by z_L. We have:

$$z_L = \left(\frac{\theta_L}{2\pi}(1+\sqrt{p})\right)^2.$$

Notice from (8) that $z_L \geq s_L$ and $z_H \leq s_H$.

For case 2a then, it suffices to show that $z_L > z_H$. In this case, there exists an $s^* \in [z_H, z_L]$ which satisfies both constraints and also lies in $[s_L, s_H]$. Comparing:

$$\begin{aligned} z_L &\geq z_H \\ \left(\frac{\theta_L}{2\pi}[1+\sqrt{p}]\right)^2 &\geq \left(\frac{\theta_H}{2\pi}[1-\sqrt{1-p}]\right)^2 \\ \frac{\theta_L}{\theta_H} &\geq \frac{1-\sqrt{1-p}}{1+\sqrt{p}}. \end{aligned} \tag{28}$$

The RHS of (28) increases monotonically from 0 to 1/2 as p goes from 0 to 1. Therefore, (28) is satisfied for all p if $\theta_L \geq (1/2)\theta_H$.

Now suppose that $\theta_L < (1/2)\theta_H$. Evaluating (28) at $p = (\theta_L/\theta_H)^2$ yields

$$\frac{\theta_L}{\theta_H} \geq \frac{1-\sqrt{1-\left(\frac{\theta_L}{\theta_H}\right)^2}}{1+\frac{\theta_L}{\theta_H}}. \tag{29}$$

Note:

$$\begin{aligned}
\left(\frac{\theta_L}{\theta_H}\right)^2 + 1 - \left(\frac{\theta_L}{\theta_H}\right)^2 &= 1 \\
\left(\frac{\theta_L}{\theta_H}\right)^2 + \sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2} &> 1 \\
\left(\frac{\theta_L}{\theta_H}\right)^2 &> 1 - \sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2} \\
\frac{\theta_L}{\theta_H} &> \frac{1 - \sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2}}{\frac{\theta_L}{\theta_H}},
\end{aligned}$$

which implies that (29) holds. Thus (28) is satisfied at $p = (\theta_L/\theta_H)^2$. But not at $p = 1$. Therefore, there exists a threshold $\tilde{p}$ such that (28) is satisfied if and only if $p \le \tilde{p}$.

Case 2b: $\Delta \in \left(1 + (1-p)\left[\theta_L/(2\pi)\right]^2, \underline{\Delta}\right]$. Now the RHS of (23) is less than the RHS of (25), so the relevant constraints are (25) and (24). (The community now fragments through zero participation not schism.)

Denote the largest s^* that satisfies (25) by $\tilde{z}_L$. We have:

$$\tilde{z}_L = \left(\frac{\theta_L}{2\pi} + \frac{1}{2}\sqrt{\left(\frac{\theta_L}{\pi}\right)^2 - 4(\Delta - 1)}\right)^2.$$

Notice from (8) that $\tilde{z}_L \ge s_L$ since $\Delta \le \underline{\Delta}$.

Hence it suffices to show that $\tilde{z}_L > z_H$. In this case, there exists an $s^* \in [z_H, \tilde{z}_L]$ which satisfies both constraints and also lies in $[s_L, s_H]$. Comparing:

$$\begin{aligned}
\tilde{z}_L &\ge z_H \\
\left(\frac{\theta_L}{2\pi} + \frac{1}{2}\sqrt{\left(\frac{\theta_L}{\pi}\right)^2 - 4(\Delta - 1)}\right)^2 &\ge \left(\frac{\theta_H}{2\pi}[1 - \sqrt{1-p}]\right)^2 \\
\theta_H\sqrt{1-p} + \pi\sqrt{\left(\frac{\theta_L}{\pi}\right)^2 - 4(\Delta - 1)} &\ge \theta_H - \theta_L. \qquad (30)
\end{aligned}$$

Note that the LHS is strictly decreasing in both Δ and p.

Evaluating (30) at $p = (\theta_L/\theta_H)^2$ and $\Delta = \underline{\Delta}$:

$$\theta_H \sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2} \geq \theta_H - \theta_L$$
$$\sqrt{1 - \left(\frac{\theta_L}{\theta_H}\right)^2} \geq 1 - \frac{\theta_L}{\theta_H}$$
$$1 - \left(\frac{\theta_L}{\theta_H}\right)^2 \geq 1 - 2\frac{\theta_L}{\theta_H} + \left(\frac{\theta_L}{\theta_H}\right)^2$$
$$\frac{\theta_L}{\theta_H} \geq \left(\frac{\theta_L}{\theta_H}\right)^2,$$

which is true since $\theta_H > \theta_L$. Evaluating (30) at $\Delta = \underline{\Delta}$ and $p = 1$ yields $0 \geq \theta_H - \theta_L$, a contradiction. Thus when Δ is at its maximum point, there exists a value $\tilde{p} \in \left([\theta_L/\theta_H]^2, 1\right)$ such that a cohesive RE exists if and only if $p \leq \tilde{p}$.

Where it exists, define $\tilde{p}(\Delta)$ as the value of p that equates the two sides of (30) for a given value of Δ. As the LHS of (30) is strictly decreasing in both Δ and p, $\tilde{p}(\Delta)$ is strictly decreasing in Δ.

At $\Delta = 1 + (1-p)\left[\theta_L/(2\pi)\right]^2$, (30) is the same as (28). By continuity of the LHS of (30), if $\theta_L > (1/2)\theta_H$, there exists a value $\hat{\Delta} \in \left(1 + (1-p)\left[\theta_L/(2\pi)\right]^2, \underline{\Delta}\right)$ such that (i) for $\Delta \leq \hat{\Delta}$ a cohesive RE exists for all p and (ii) for $\Delta > \hat{\Delta}$ a cohesive RE exists if and only if $p \leq \tilde{p}(\Delta)$. If $\theta_L < (1/2)\theta_H$, for all $\Delta \in \left(1 + (1-p)\left[\theta_L/(2\pi)\right]^2, \underline{\Delta}\right]$, a cohesive RE exists if and only if $p \leq \tilde{p}(\Delta)$.

Schismatic RE. The following conditions are necessary and sufficient. To rule out emergence of an inclusive club, there must not exist an $s^* \in [0,1]$ such that (23) and (24) hold.

In addition, there are the participation constraints

$$\pi \left[1 + (1-p)\left(\frac{\theta_L}{2\pi}\right)^2\right] \geq \Delta\pi, \tag{31}$$
$$\pi \left[1 + p\left(\frac{\theta_H}{2\pi}\right)^2\right] \geq \Delta\pi. \tag{32}$$

For $\Delta \leq \underline{\Delta}$ and $p < (\theta_L/\theta_H)^2$, we have established that a club can break up a schismatic state and form an inclusive club at $s^* = s_L$. For $\Delta > 1 + (1-p)\left[\theta_L/(2\pi)\right]^2$, clearly (31) is violated. That leaves $\Delta \leq 1 + (1-p)\left[\theta_L/(2\pi)\right]^2$ and $p \geq (\theta_L/\theta_H)^2$. For such Δ, (31) is satisfied. For such p, the LHS of (31) is less than the LHS of (32). Hence (32) is also satisfied. We established in case 2a above that either (23) or (24) are violated for all $s^* \in [0,1]$ if and only if $p \geq \tilde{p}$, where $\tilde{p}$ equates (28). Therefore, a schismatic RE exists for $\Delta \leq 1 + (1-p)\left[\theta_L/(2\pi)\right]^2$ and $p \geq \tilde{p}$.

Exclusive RE. The conditions are the same as for a schismatic RE except that the weak inequality in (31) is reversed. Hence an exclusive RE exists for $\Delta \leq \underline{\Delta}$ wherever neither a cohesive or schismatic RE exists. □

Proof of Proposition 4

Proof. Define X_M as total participation under monopoly and X_C as total participation under competition.

Recall that each club k maximizes X_k. A monopolist can recreate any competitive RE on its own except the schismatic one. Hence $X_M \geq X_C$ whenever an inclusive or exclusive club is formed under competition.

The inequality is strict for an open set of parameters. For example, the RE is cohesive in both the monopoly and competitive cases when $p < (\theta_L/\theta_H)^2$ and $\Delta < 1$. In such a case, $\underline{s} > s^*$.

In addition, we claim that $X_M \geq X_C$ whenever the competitive RE is schismatic. Taken together, this would establish the proposition. Let us now prove the claim.

Case 1: The RE under monopoly is exclusive at strictness $\overline{s}(p)$, whereas schism occurs under competition with strictness levels $\tilde{s}_L$ and $\tilde{s}_H$.

Total participation is no less under monopoly if

$$\begin{aligned} X_C(p,\delta) &\leq X_M(p,\delta) \\ (1-p)\tilde{s}_L + p\tilde{s}_H &\leq p\overline{s}(p). \end{aligned}$$

Recall from Proposition 3 that schism occurs only when $\Delta \in (\theta_L/\pi, \tilde{\Delta}]$, where $\tilde{\Delta} \equiv 1 + (1-p)\,(\theta_L/2\pi)^2$. Hence the RHS is minimized at $\Delta = \tilde{\Delta}$, in which case $\overline{s}(p) = (\sqrt{\tilde{s}_H} + \sqrt{\tilde{s}_H - \tilde{s}_L})^2$ (see (4)). Thus it is sufficient to verify

$$\begin{aligned} (1-p)\tilde{s}_L + p\tilde{s}_H &\leq p\left(\sqrt{\tilde{s}_H} + \sqrt{\tilde{s}_H - \tilde{s}_L}\right)^2 \\ (1-p)\tilde{s}_L + p\tilde{s}_H &\leq p\left(\tilde{s}_H + \tilde{s}_H - \tilde{s}_L + 2\sqrt{\tilde{s}_H(\tilde{s}_H - \tilde{s}_L)}\right) \\ \tilde{s}_L &\leq p\left(\tilde{s}_H + 2\sqrt{\tilde{s}_H(\tilde{s}_H - \tilde{s}_L)}\right). \end{aligned} \tag{33}$$

Multiplying both sides by $[p/(1-p)](1/\tilde{s}_H)$ yields

$$\left(\frac{\theta_L}{\theta_H}\right)^2 \leq \frac{p^2}{1-p} + \frac{2p}{1-p}\sqrt{\frac{\tilde{s}_H - \tilde{s}_L}{\tilde{s}_H}}.$$

Where schism occurs, $\tilde{s}_H > \tilde{s}_L$. Hence the second term on the right-hand side is nonnegative. Therefore it suffices that

$$\left(\frac{\theta_L}{\theta_H}\right)^2 \leq \frac{p^2}{1-p}. \tag{34}$$

We also know from Proposition 3 that $p > \tilde{p}$ where schism occurs. The right-hand side of (34) is strictly increasing in p. Therefore, if the inequality holds at $\tilde{p}$, it holds for all $p > \tilde{p}$. Given that schism occurs, $2\theta_L < \theta_H$, so by the proof of Proposition 3, $\tilde{p}$ is defined implicitly by

$$\frac{\theta_L}{\theta_H} = \frac{1 - \sqrt{1-\tilde{p}}}{1 + \sqrt{\tilde{p}}}. \tag{35}$$

Substituting (35) into (34), it remains to show that

$$\begin{aligned}\left(\frac{1-\sqrt{1-\tilde{p}}}{1+\sqrt{\tilde{p}}}\right)^2 &\leq \frac{\tilde{p}^2}{1-\tilde{p}} \\ \frac{1-\sqrt{1-\tilde{p}}}{1+\sqrt{\tilde{p}}} &\leq \frac{\tilde{p}}{\sqrt{1-\tilde{p}}} \\ \sqrt{1-\tilde{p}}-(1-\tilde{p}) &\leq \tilde{p}+\tilde{p}^{\frac{3}{2}} \\ \sqrt{1-\tilde{p}} &\leq 1+\tilde{p}^{\frac{3}{2}},\end{aligned}$$

which is true for all $\tilde{p} \in [0,1]$.

Case 2: The RE is cohesive under monopoly and schismatic under competition.

Therefore, total participation is no less under monopoly if

$$\underline{s} \geq (1-p^t)\tilde{s}_L + p^t\tilde{s}_H.$$

Because schism occurs, we know $p > \tilde{p}$. In *case 1*, we showed for $p > \tilde{p}$ that

$$p\overline{s}(p) \geq (1-p)\tilde{s}_L + p\tilde{s}_H.$$

As an inclusive club forms under monopoly, we know $\underline{s} \geq p\overline{s}(p)$. Hence

$$\underline{s} \geq p\overline{s}(p) \geq (1-p)\tilde{s}_L + p\tilde{s}_H.$$

This establishes the claim and the proposition. □